



## Cambridge International AS & A Level

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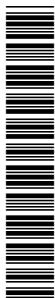
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**MATHEMATICS**

**9709/13**

Paper 1 Pure Mathematics 1

**May/June 2021**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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- 1 A curve with equation  $y = f(x)$  is such that  $f'(x) = 6x^2 - \frac{8}{x^2}$ . It is given that the curve passes through the point  $(2, 7)$ .

Find  $f(x)$ .

[3]

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2 The function  $f$  is defined by  $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$  for  $\frac{1}{2} < x < a$ . It is given that  $f$  is a decreasing function.

Find the maximum possible value of the constant  $a$ . [4]

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3 A line with equation  $y = mx - 6$  is a tangent to the curve with equation  $y = x^2 - 4x + 3$ .

Find the possible values of the constant  $m$ , and the corresponding coordinates of the points at which the line touches the curve. [6]

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- 4 (a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where  $k$  is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k. \quad [2]$$

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- (b) Hence express  $\cos x$  in terms of  $k$ . [2]

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- (c) Hence solve the equation  $\frac{\tan x + \sin x}{\tan x - \sin x} = 4$  for  $-\pi < x < \pi$ . [2]

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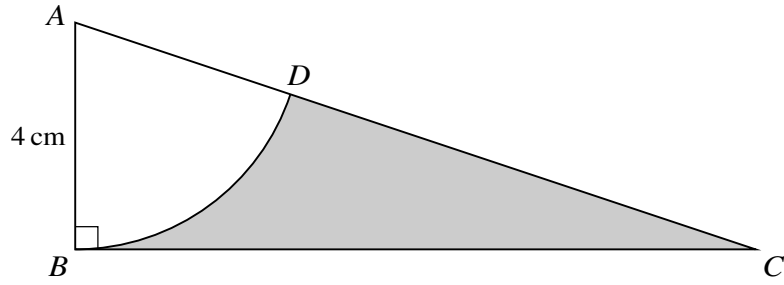
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The diagram shows a triangle  $ABC$ , in which angle  $ABC = 90^\circ$  and  $AB = 4$  cm. The sector  $ABD$  is part of a circle with centre  $A$ . The area of the sector is  $10 \text{ cm}^2$ .

(a) Find angle  $BAD$  in radians. [2]

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(b) Find the perimeter of the shaded region. [4]

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6 Functions  $f$  and  $g$  are both defined for  $x \in \mathbb{R}$  and are given by

$$f(x) = x^2 - 2x + 5,$$

$$g(x) = x^2 + 4x + 13.$$

- (a) By first expressing each of  $f(x)$  and  $g(x)$  in completed square form, express  $g(x)$  in the form  $f(x + p) + q$ , where  $p$  and  $q$  are constants. [4]

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- (b) Describe fully the transformation which transforms the graph of  $y = f(x)$  to the graph of  $y = g(x)$ . [2]

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- 7 (a) Write down the first four terms of the expansion, in ascending powers of  $x$ , of  $(a - x)^6$ . [2]

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- (b) Given that the coefficient of  $x^2$  in the expansion of  $\left(1 + \frac{2}{ax}\right)(a - x)^6$  is  $-20$ , find in exact form the possible values of the constant  $a$ . [5]

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10 Points  $A(-2, 3)$ ,  $B(3, 0)$  and  $C(6, 5)$  lie on the circumference of a circle with centre  $D$ .

(a) Show that angle  $ABC = 90^\circ$ . [2]

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(b) Hence state the coordinates of  $D$ . [1]

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(c) Find an equation of the circle. [2]

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The point  $E$  lies on the circumference of the circle such that  $BE$  is a diameter.

(d) Find an equation of the tangent to the circle at  $E$ . [5]

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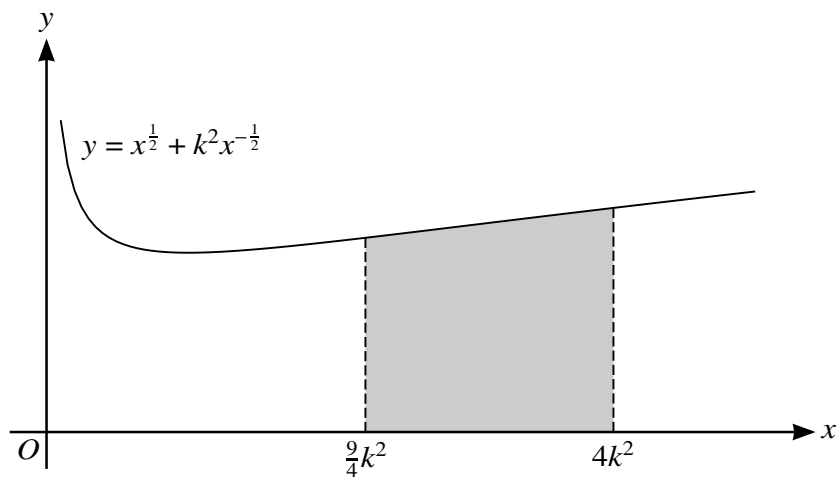
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The diagram shows part of the curve with equation  $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$ , where  $k$  is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of  $k$ . [4]

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The tangent at the point on the curve where  $x = 4k^2$  intersects the  $y$ -axis at  $P$ .

(b) Find the  $y$ -coordinate of  $P$  in terms of  $k$ . [4]

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The shaded region is bounded by the curve, the  $x$ -axis and the lines  $x = \frac{9}{4}k^2$  and  $x = 4k^2$ .

(c) Find the area of the shaded region in terms of  $k$ . [3]

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**Additional Page**

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